Velocity Defect Law for a Transpired Turbulent Boundary Layer

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Recent experiments in our laboratory indicate that Clauser's[1] concept of the turbulent boundary layer formed on an impervious surface may be extended to the transpired turbulent boundary layer. The experimental equipment is described in detail elsewhere.[2] Two isothermal, air-to-air transpiration runs were performed, both at a constant mainstream velocity of 25 fps. The ratio of superficial injection velocity to mainstream velocity (V_0/U_1) was 1×10^{-3} and 3×10^{-3}, respectively, for the two runs. The shear stress profiles were determined from the measured mean velocity profiles by hand computation employing momentum integral-type expressions. It was found that the shear stress rose rapidly from its wall value, soon reached a broad, well-defined maximum in the region $0.1 < y/\delta < 0.2$, and then decreased monotonically to zero at $y/\delta = 1$.

The velocity defect law applies only to the outer portion of the boundary layer, and a friction velocity based on the wall shear stress is normally employed as the scale velocity. Coles'[3,4] empirical correlation for a nontranspired boundary layer is

$$\frac{U_1 - U}{U_\tau} = -\frac{1}{K}\ln\frac{y}{\delta} + \frac{\pi(x)}{K}\left[2 - w\left(\frac{y}{\delta}\right)\right] \tag{1}$$

However, it is obvious from the preceding paragraph that the wall shear stress does not characterize adequately the processes occurring in the outer portion of the transpired boundary layer. Moreover, Clauser's model envisages an outer region of constant eddy viscosity that "floats" on a complicated substrate to which it is coupled only loosely. In view of these facts, it seems appropriate to employ a friction velocity $U_\tau{}^*$, based on the *maximum* shear stress, as the scale velocity. For flows with no transpiration and no axial pressure gradient, the maximum shear stress occurs at the wall, so this is perfectly consistent with current practice. Equation (1) then becomes

$$\frac{U_1 - U}{U_\tau{}^*} = -\frac{1}{K}\ln\frac{y}{\delta} + \frac{\pi(x)}{K}\left[2 - w\left(\frac{y}{\delta}\right)\right] \tag{2}$$

Furthermore, it is expected that the gross structure of the outer portion will be independent of transpiration rate. Hence, K, $\pi(x)$, and $w(y/\delta)$ should be unchanged by transpiration. The authors have employed the values most recently recommended by Coles[4] [$K = 0.41$ and $\pi(x) = 0.55$].

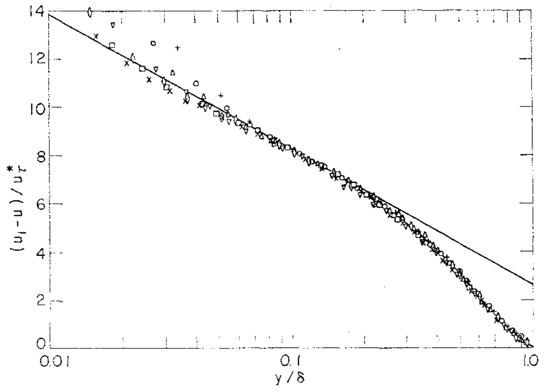

Fig. 1 $V_0/U_1 = 1 \times 10^{-3}$

+ Re_θ = 831
○ Re_θ = 1047
△ Re_θ = 1293
∇ Re_θ = 1506
◊ Re_θ = 1914
□ Re_θ = 2332
X Re_θ = 2654

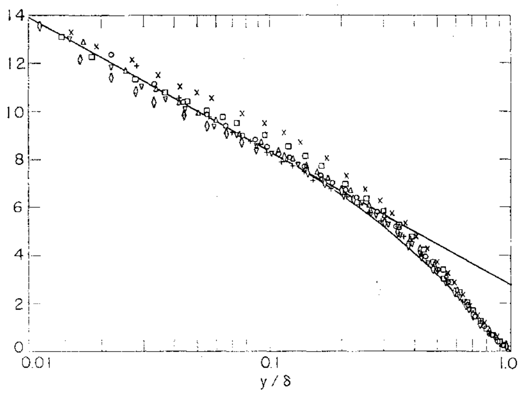

Fig. 2 $V_0/U_1 = 3 \times 10^{-3}$

+ Re_θ = 1170
○ Re_θ = 1509
△ Re_θ = 2030
∇ Re_θ = 2198
◊ Re_θ = 2980
□ Re_θ = 3701
X Re_θ = 4411

Figure 1 shows a comparison of Eq. (2) with the data obtained for $V_0/U_1 = 1 \times 10^{-3}$. Over the outer 90% of the boundary layer, the agreement is excellent. Figure 2 is the corresponding plot for $V_0/U_1 = 3 \times 10^{-3}$. In this case, the agreement is not quite as good, but it is well within the precision of the data.

These incomplete results indicate that Clauser's concept of a simple outer region "floating" on a complicated substrate is an extremely powerful one that may have real significance for transpired boundary layers. Currently the range and type of experimental measurements are being extended in order to examine the concept in more detail.

References

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[3] Coles, D., "The law of the wake in the turbulent boundary layer," J. Fluid Mech. 1, Part 2, 191–226 (1956).

[4] Coles, D., "The turbulent boundary layer in a compressible fluid," The Rand Corp., Santa Monica, Calif. (1961).

Received April 19, 1963.

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Simple Method of Analyzing Dissociative and Vibrational Relaxation behind Oblique Shock Waves

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Recently, in several theoretical studies of dissociated boundary-layer flows with gas phase and/or catalytic surface reactions,[1–3] there has been exploited a remarkably simple yet accurate approximate method of predicting the nonequilibrium flow properties for arbitrary values of the reaction rate. The method consists of performing a local, nonlinear extrapolation of the exact first-order (nearly frozen)

Received April 22, 1963.

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